

1. **THE PROBLEM**
 The problem is to determine the value of the function $f(x)$ for all real numbers x . The function is defined by the following properties:

- $f(0) = 1$
- $f(x+y) = f(x)f(y)$ for all real numbers x and y .
- $f(x) > 0$ for all real numbers x .

2. **SOLUTION**
 We will show that $f(x) = e^{kx}$ for some constant k . First, let $x = y = 0$. Then $f(0) = f(0)f(0)$, which implies $f(0) = 1$ or $f(0) = 0$. Since $f(0) = 1$ by property 1, we have $f(0) = 1$. Next, let $y = -x$. Then $f(x) = f(x)f(-x)$, which implies $f(-x) = 1/f(x)$. This shows that $f(x)$ is never zero. Now, let $x = y = 1$. Then $f(2) = f(1)f(1) = f(1)^2$. By induction, we can show that $f(n) = f(1)^n$ for all positive integers n . Similarly, $f(-n) = 1/f(n) = 1/f(1)^n = f(1)^{-n}$. For rational numbers $r = p/q$, we have $f(r) = f(1)^r$. Finally, by continuity, we can extend this to all real numbers x , showing that $f(x) = e^{kx}$ for some constant k .

3. **CONCLUSION**
 The function $f(x)$ is uniquely determined by the given properties and is given by $f(x) = e^{kx}$ for some constant k . The value of k depends on the specific function being considered.